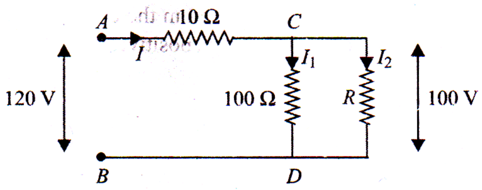
WEEKLY TEST TARGET - JEE - TEST - 23
SOLUTION Date 20-10-2019

[PHYSICS]

1. Potential difference across C and D is 100 V.



Hence, $I_1 = \frac{100}{100} = 1\text{A}$, $V_{AC} = 120 - 100 = 20$ V

And $I = \frac{20}{10} = 2\text{A}$. Hence, $I_2 = 2 - 1 = 1$ A

$R = 100/I_2 = 100\ \Omega$

2. $\vec{F} = m\frac{\Delta\vec{v}_{cm}}{\Delta t} = \frac{10(3\hat{j} - 4\hat{i})}{2} = 5(3\hat{j} - 4\hat{i})$

$|\vec{F}| = 5\sqrt{3^2 + 4^2} = 25N$

3. Current through 1 Ω resistance will be 2 A in the upward direction.
$V_G - 2\times4 + 3 - 2\times2 + 2\times1 = V_H$ or $V_G - V_H = 7$ V

4. Let the speeds of balls of mass m and $2m$ after collision be v_1 and v_2 as shown in the figure. Applying conservation of momentum

$$mv_1 + 2mv_2 = mu \text{ and } -v_1 + v_2 = \frac{u}{2}$$

Solving we get $v_1 = 0$ and $v_2 = \frac{u}{2}$

Hence, the ball of mass m comes to rest and ball of mass $2m$ moves with speed.

$$t = \frac{2\pi r}{u/2} = \frac{4\pi r}{u}$$

5. Evidently, when the block B descends by a distance 'l'. The pulley P, descends by a distance $\frac{\ell}{2}$, and consequently, the block A ascends by a distance $\frac{\ell}{2}$. Also, if 'v' be the speed of block A, (up), $2v$, would be the speed of block 'B' (down).
Net loss in P.E. of the system = Loss in P.E of block B. – gain in P.E. of block A

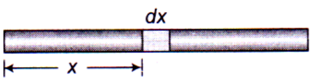
$$mg\frac{\ell}{2} = \frac{5}{2}mv^2 \Rightarrow v^2 = \frac{g\ell}{5}$$

Total gain in K.E. $= \frac{1}{2}mv^2 + \frac{1}{2}m(2v)^2 = \frac{5}{2}mv^2$

Now, law of conservation of energy demands
Loss in P.E. = Gain in K.E.

$$mg\frac{\ell}{2} = \frac{5}{2}mv^2 \Rightarrow v^2 = \frac{g\ell}{5}$$

$$v = \sqrt{\frac{g\ell}{5}}$$

6. 

$$x_{cm} = \frac{\int_0^L \frac{K}{L}x^2 dx.x}{\int_0^L \frac{K}{L}x^2 dx} = \frac{\left.\frac{x^4}{4}\right|_0^L}{\left.\frac{x^3}{3}\right|_0^L} = \frac{3}{4}L$$

7. The mass move under the influence of gravitational pull which acts along the vertical. Thus CM changes along vertical while it remains unchanged in the horizontal direction.

8. Initial velocity of the particle is $v_1 = 20$ m/s
Final velocity of the particle is $v_f = 0$
From work energy theorem

$$W_{net} = \Delta K.E. = K_f - K_i$$
$$= \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(2)(0-400) = -400 \text{ J}$$

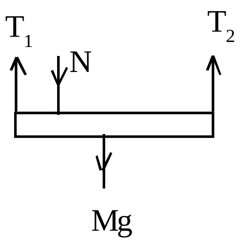
9.
10.
11.
12.
13.
14. [C] Conserving angular momentum about point P

$$\frac{ML^2}{12}\omega = \frac{ML^2}{3}\omega_1; \quad \omega_1 = \frac{\omega}{4}; \quad \therefore v_B = \frac{L\omega}{4}$$

15. [C] $T_1\frac{3}{2} = T_2\frac{3}{2} + N\frac{1}{2}$

$N + T_2 = Mg$
$T_1 + T_2 = N + Mg$

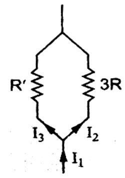
$$\frac{T_1}{T_2} = \frac{6}{5}$$



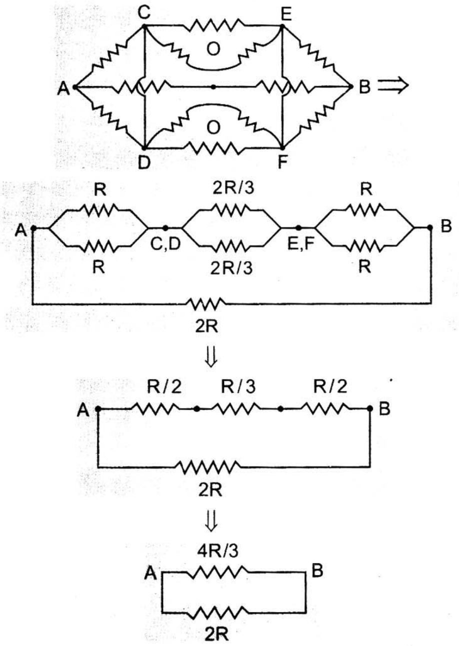
16. . Given $I_2 = I_3$

$I_2 \times 3R = I_3 R' = I_2 R'$

So, $R' = 3R$



17. The current in CO, OE, DO and OF will be same. So, these branches will not touch each other at O.



$$R_{eq} = \frac{8}{10} R.$$

18. No current will pass through C_1 and C_2

$R_{eq} = 10\Omega$

$I = 1\text{A}$

$V_A - V_C = 1 \times 6 = 6\text{V}$

$V_C = V_A - 6 = 4\text{V}$

$V_C - V_E =$ voltage difference across

$C_2 = 4\text{V}$.

19. It is the speed of light in free space. Hence, dimension is that of speed, *i.e.*, LT^{-1}.

20. Distance travelled in nth second,

$$s_n = u + a\left(n - \frac{1}{2}\right)$$

Distance travelled in 2nd second,

$$s_2 = 0 + a\left(2 - \frac{1}{2}\right) \qquad \ldots(i)$$

Distance travelled in 5th second,

$$s_5 = 0 + a\left(5 - \frac{1}{2}\right) \qquad \ldots(ii)$$

Dividing eqn. (ii) by eqn. (i), $s_5 = 24$ m

21.

$$v = \sqrt{v_0^2 + 2gy}$$

and v sin θ = v_0 (as given in question)

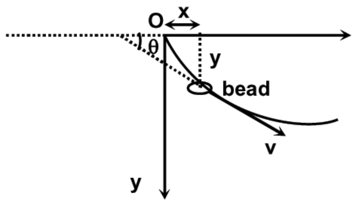
$$\therefore \ \sin\theta = \frac{v_0}{\sqrt{v_0^2 + 2gy}}$$

$$\therefore \tan\theta = \frac{v_0}{\sqrt{2gy}}$$

$$\frac{dy}{dx} = \frac{v_0}{\sqrt{2gy}}$$

$$\therefore \ y = \frac{(3gv_0x)^{2/3}}{2g}$$

$\therefore$ a + b + c = 8

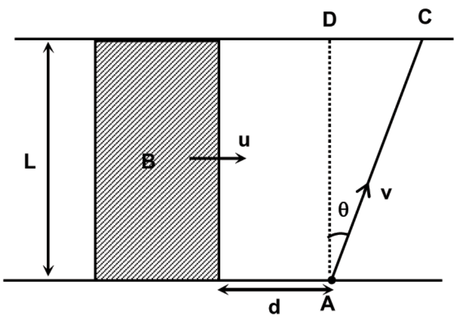


22. Time to cross, $t = \dfrac{L}{v\cos\theta}$

$$\therefore \ ut = \ d + L\tan\theta = \frac{uL}{v\cos\theta}$$

$$\therefore \ v = \frac{uL}{d\cos\theta + L\sin\theta}$$

$$\therefore \ v_{min} = \frac{uL}{\sqrt{d^2 + L^2}} = 4 \ \text{m/s}$$



23. If they meet a height h after time T of the projection of the second.

Then, $h = u(T) - \frac{1}{2}g(T)^2 = v(T-t) - \frac{1}{2}g(T-t)^2 \quad \ldots(i)$

$$T = \frac{5t^2 + 3t}{10t - 2}$$

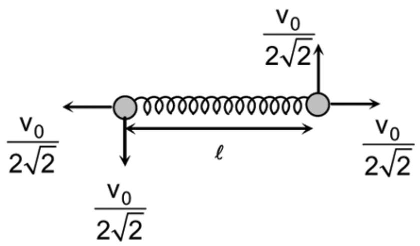
For minimum T, $\dfrac{dT}{dt} = 0$

$50t^2 - 20t - 6 = 0$

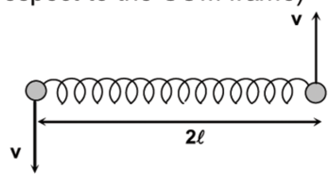
$\Rightarrow t = 0.6 = \dfrac{6}{g}$

24. For car + trailer system
$P_{max} = (3mg \sin\theta + 3kmv^2)v$
For car only
$P_{max} = (mg \sin\theta + 4kmv^2)2v$
$\therefore$ $(3mg \sin\theta + 3kmv^2)v = (mg \sin\theta + 4kmv^2)2v$
$\therefore\ k = \dfrac{g\sin\theta}{5v^2}$
For down the incline $kmv_t^2 = mg \sin\theta$
$\therefore\ v_t^2 = 5v^2$

25. At the starting (with respect to COM frame)



At the instant of maximum elongation (with respect to the COM frame)



Conservation of angular momentum with respect to centre of mass (COM) frame
$$2m\frac{v_0}{2\sqrt{2}}\frac{\ell}{2} = 2mv\ell$$
$$\therefore\ v = \frac{v_0}{4\sqrt{2}}$$
From conservation of energy with respect to centre of mass frame
$$2\frac{1}{2}m\left(\frac{v_0^2}{8}+\frac{v_0^2}{8}\right) - 2\frac{1}{2}m\frac{v_0^2}{32} = U_{spring}$$
$$\therefore\ U_{spring} = mv_0^2\left(\frac{1}{4}-\frac{1}{32}\right) = \frac{7mv_0^2}{32} = 2 \text{ Joule}$$

[CHEMISTRY]

26. [c] suppose the formula of hydrocarbon is C_xH_y.
$$C_xH_y + \left(x\frac{y}{4}\right)o_2 \rightarrow xco_2 + \frac{y}{2}H_2O$$
1 vol x + y/4 vol. x vol 0 vol

20 ml 20 $(x + \frac{y}{4})$ ml 20 ml 0

From question vol . of co_2 formed = 60 ml
20 x = 60 $\therefore$ x = 3
vol of o_2 used = 100 ml

$20\,(x + \frac{y}{4}) = 100$ $\therefore$ y = 8
Hence hydrocarbon is C_3H_8

27. [c] neq . of $KMNO_4$ = neq of $H_2C_2O_4$

$$\frac{v\times 0.1\times 5}{1000}=\frac{40\times 0.2\times 2}{1000}$$

$v = 32$ ml

28. [d] fact

29. [c] fact

30. 6 e^- in valence shell of boron

31. $$B_2H_6 + NH_3 \xrightarrow[\text{low temperature}]{\text{Excess } NH_3} B_2H_6 . 2NH_3$$

or $[H_2B(NH_3)_2]^+ [BH_4]^-$ (ionic compound).

32 $$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

$$(Sn^{2+} \rightarrow Sn^{4+} + 2e^-) \quad 3$$

$$Cr_2O_7^{2-} + 14H^+ + 3Sn^{2+} \rightarrow 3Sn^{4+} + 2Cr^{3+} + 7H_2O$$

It is clear from this equation that 3 moles of Sn^{2+} reduce one mole of CrO_7^{2-}, hence 1 mol of Sn^{2+} will reduce $\frac{1}{3}$ moles of CrO_7^{2-}.

33. (a) $$\underset{+1}{K}\,\underset{+5}{Cl}\,\underset{-2}{O_3} \longrightarrow \underset{+1}{K}\,\underset{-1}{Cl} + 3\underset{0}{O_2}$$

Cl is reduced and O is oxidised.

(b) $$(\underset{-3}{N}H_4)_2\,\underset{+6}{Cr_2}O_7 \longrightarrow \underset{0}{N_2} + \underset{+3}{Cr_2}O_3$$

N is oxidised and Cr is reduced.

34. Scattering of light; reason for blue colour for sky.

35. $LiHCO_3$ is unstable and exists only in solution.

36. LiCl being covalent has the lowest melting point. The melting points of other halides decrease form NaCl to CsCl as the lattice energies decrease.

37. $BaCO_3 > SrCO_3 > CaCO_3 > MgCO_3$. Thermal stability decreases as the basic character of the metal hydroxide decreases.

38. MgO is basic whereas all other three oxides are amphoteric in nature.

39. $$\frac{(r_2)Li^+}{(r_3)He^+}=\frac{0.529\times 2^2/3}{0.529\times 3^2/2}=\frac{8}{27}$$

40. .[b] Fact

41. [b] $CaCl_2 + NaCl \rightarrow$ $$CaCO_3 \xrightarrow{\Delta} CaO$$

xg $(10 - x)_g$

$\therefore$ g m eq. of cao = gm eq. of $CaCl_2$

$$\frac{1.62}{56} = \frac{x}{111}$$

$\therefore$ mass of $CaCl_2 = 3.21$ g

or % by mass = 32.1 %

42. [a] each possesses 14 e – 5 with board order 3.

43. [a] $NH_3 + BF_3 \rightarrow H_3N \rightarrow BF_3$

sp^3 sp^2 sp^3 sp^3

44. [c] O_2^- has one unpaired e

45. (a) Work done = Area enclosed by triangle

ABC $= \frac{1}{2} AC \times BC = \frac{1}{2} \times (3V - V) \times (3P - P) = 2PV$

46. $\lambda = \frac{2\pi r_n}{n}$

$$\lambda = \frac{2 \times 3.14 \times 0.529 \times 10^{-10} \times n^2}{n}$$

$1.67 \times 10^{-9} = 2 \times 3.14 \times 0.529 \times 10^{-10} \times n$

$\therefore n = 5$

47. $2Cr(OH)_3 + IO_3^- + 4OH^- \longrightarrow 2CrO_4^{2-} + I^- + 5H_2O$

x = 2, y = 1, z = 4

$\therefore x + y + z = 2 + 1 + 4 = 7$

48. $\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{P_2}{P_1}\right)^{n-1}$ n – 1 = 1

$\frac{10}{5} = \left(\frac{200}{100}\right)^{n-1}$ n = 2

$2 = 2^{n-1}$

49. Boron Nitride, Boric acid, Beryllium hydride, Graphite.

50. $[CH_3COOH] = C_1 = \frac{200 \times 1}{400} = 0.5$

$[HCOOH] = C_2 = \frac{200 \times 0.1}{400} = 0.05$

$[H^+] = \sqrt{K_{a_1}C_1 + K_{a_2}C_2}$

$= \sqrt{(10^{-6} \times 0.5) + (10^{-5} \times 0.05)}$

$= 10^{-3}$

pH = - log 10^{-3} = 3

[MATHEMATICS]

51. $y - e^{xy} + x = 0$

Differentiating w.r.t. to y

$$1 - e^{xy}\left(\frac{dx}{dy} \cdot y + x\right) + \frac{dx}{dy} = 0$$

$$\frac{dx}{dy} = 0$$

$1 - xe^{xy} = 0$

$xe^{xy} = 1 \quad \Rightarrow x = 1, y = 0$

$\therefore$ Point is (1, 0)

52. $T_{r+1} = {}^{6561}C_r (7)^{\frac{6561-r}{3}} \left(11^{\frac{1}{9}}\right)^r$

Here r should be a multiple of 9

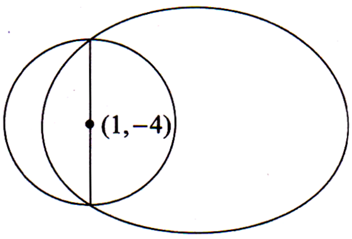
$r = 0, 9, 18 .... 6561$

Number of terms = 730

53. Common chord of given circle

$6x + 4y + (p + q) = 0$

This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$



centre (1, –4)

$6 - 16 + (p + q) = 0 \quad \Rightarrow \quad p + q = 10$

54. $\text{Det.}(2A^9B^{-1}) = \frac{2^2(\text{Det.}A)^9}{\text{Det.}B} = \frac{2^2(-1)^9}{2}$

$= -2$

55. **Hint:** for continuity at $x = 0$

$\lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(0-h) = f(0)$

$\lim_{h\to 0} e^{-h} + a = -3 \quad \Rightarrow \quad a = -4;$

For the value of a, f is diff at $x = 0$

56. $\prod_{r=1}^{S_n} \begin{bmatrix} 1 & 2r-1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & 99 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1+3+5+7+\cdots+99 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2500 \\ 0 & 1 \end{bmatrix}$

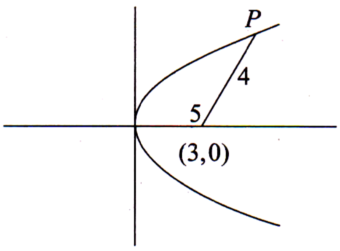
57. $f(x) = \log_x (lnx)$

$\Rightarrow \quad f(x) = \dfrac{ln(lnx)}{lnx}$

$\Rightarrow \quad f'(x) = \dfrac{lnx\left(\frac{1}{lnx}\cdot\frac{1}{x}\right) - \frac{1}{x}ln(lnx)}{(lnx)^2}$

$\Rightarrow \quad f'(e) = \dfrac{\frac{1}{e} - 0}{1} = \dfrac{1}{e}$

58.



Let the point P be $(3t^2, 6t)$
and $PS = 3 + 3t^2 = 4$

$t^2 = \dfrac{1}{3}$

$t = \pm\dfrac{1}{\sqrt{3}}$

$\therefore$ Points are
$(1, 2\sqrt{3})$ and $(1, -2\sqrt{3})$

59. $x + \dfrac{100}{x} > 50$

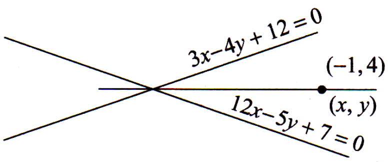
$x = 1$ Satisfies, $= 2$ Satisfies — 2 numbers

Ans $P = \dfrac{55}{100}$

$= 3$ does not satisfy
$= 47$ does not satisfy

$= 48$ Satisfies, $= 100$ Satisfies — 53 numbers

60. At $(-1, 4)$



$3x - 4y + 12 < 0$ and $12x - 5y + 7 < 0$

$\Rightarrow \quad \dfrac{3x - 4y + 12}{12x - 5y + 12} > 0$ at $(-1, 4)$

So we have to take the bisector with + sign

$$\frac{3x-4y+12}{5}=\frac{12x-5y+7}{13}$$

$$21x+27y-121=0$$

61.

A

Given Expression

$$=\sin^{-1}\cot\left\{\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$=\sin^{-1}\cot\left\{\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}\right\}$$

$$=\sin^{-1}\left\{\cot\frac{\pi}{2}\right\} \quad =\sin^{-1}0=0$$

62.

B

Let $y=\frac{e^{2x}-1}{e^{2x}+1}$

$\Rightarrow e^{2x}=\frac{-(y+1)}{(y-1)}>0$

$\Rightarrow -1<y<1$

63.

B

$3\sin P+4\cos Q=6$ ....(1)

$4\sin Q+3\cos P=1$ ....(2)

From(1) and (2)P is obtuse.

$(3\sin P+4\cos Q)^2+(4\sin Q+3\cos P)^2=37$

$\Rightarrow$ $9+16+24(\sin P\cos Q+\cos P\sin Q)=37$

$\Rightarrow$ $24\sin(P+Q)=12$

$\Rightarrow$ $P+Q=\frac{5\pi}{6}$

$\Rightarrow$ $R=\frac{\pi}{6}$

64.

C

Here

$R=\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$

$\therefore$ $R^{-1}=\{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$

Hence

$R \text{ o } R^{-1}=\{(3,3),(3,5),(5,3),(5,5)\}$

65.

C

We have

$$4\cot 2\theta = \cot^2\theta - \tan^2\theta$$

$$\Rightarrow \quad \frac{4}{\tan 2\theta} = \frac{4}{\tan^2\theta} - \tan^2\theta$$

Put $\quad \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\therefore \quad \frac{4(1-\tan^2\theta)}{2\tan\theta} = \frac{1-\tan^4\theta}{\tan^2\theta}$$

$$\Rightarrow \quad (1-\tan^2\theta)\left[2\tan\theta - (1+\tan^2\theta)\right] = 0$$

$$\Rightarrow \quad (1-\tan^2\theta)(\tan^2\theta - 2\tan\theta + 1) = 0$$

$$\Rightarrow \quad (1-\tan^2\theta)(\tan\theta - 1)^2 = 0$$

$$\Rightarrow \quad \tan\theta = 1, -1 \quad \therefore \quad \theta = n\pi \pm \frac{\pi}{4}$$

66.

C

We have

$$|z+\bar{z}| + |z-\bar{z}| = 8$$

$$\Rightarrow \quad 2|x| + 2|y| = 8 \quad \text{or} \quad |x| + |y| = 4$$

67.

B

Since $x^2 - 3x + 2$ is one of the factors of the expression $x^4 - px^2 + q$, therefore, on dividing the expression by factor, remainder =0 i.e., on dividing $x^4 - px^2 + q$ by $x^2 - 3x + 2$, the remainder

$(15 - 3p)x + (2p + q - 14) = 0$

On comparing both sides, we get

$15 - 3p = 0$ or $p\ 5$

and $2p + q - 14 = 0$ or $q\ 4$

68.

C

$\det(B^{-1}AB) = \det(B^{-1})\det A \det B$

$= \det(B^{-1}) \times \det B \times \det A = \det(B^{-1}B) \times \det A$

$= \det(I) \times \det A = 1 \times \det A = \det A.$

69.

D

$$\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+2a-3 & 2a-2 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$(\text{Applying } R_1 \to R_3, \text{ and } R_2 \to R_2 - R_3)$

$$= \begin{vmatrix} a^2+2a-3 & 2a-2 \\ 2a-2 & a-1 \end{vmatrix}$$

$(\text{Expanding along } C_3)$

$$= (a-1)^2 \begin{vmatrix} a+3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (a-1)^2.(a+3-4) = (a-1)^3$$

Clearly, $\Delta > 0$ if $a > 1 : \Delta = 0$ if $a = 1$ and $\Delta < 0$ if $a < 1$

70.

B

Given

$P(A) = 0.5, P(B) = 0.3$ and $P(C) = 0.2$

$\therefore \quad P(A) + P(B) + P(C) = 1$

So the events A, B, C are exhaustive.

If $P(E)$ = Probability of introducing a new product, then as given

$P(E/A) = 0.7, P(E/B) = 0.6$ and $P(E/C) = 0.5$

$\therefore \quad P(E) = P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right) + P(C).P\left(\frac{E}{C}\right)$

$= 0.5 \times 0.7 + 0.3 \times 0.6 + 0.2 \times 0.5$

$= 0.35 + 0.18 + 0.10 = 0.63$

71.

C

Required limit

$$= \lim_{x\to 0} \frac{243^x(3^x-1)-9^x(3^{2x}-1)+(3^x-1)}{x^3}$$

$$= \lim_{x\to 0} \frac{(3^x-1)\{(243)^x-(27)^x-9^x+1\}}{x^3}$$

$$= \lim_{x\to 0} \frac{(3^x-1)\{(243)^x-(27)^x-9^x+1\}}{x^3}$$

$$= \lim_{x\to 0} \frac{(3^x-1)(9^x-1)(27^x-1)}{x \quad x \quad x}$$

$= \log 3 . \log 9 . \log 27$

$= \log 3 . 2\log 3 . 3\log 3$

$= 6(\log 3)^3 = k(\log 3)^3$

$k = 6$

72. For least area $\lambda = 0$

So; $A_{min} = 2\int_0^4 \left(12-\left(x^2-4\right)\right)dx = \frac{256}{3}$

73. Let $f(x) = \int_0^x \frac{t^8+1}{t^8+t^2+1}dt - 3x + 2$

f(0) = 2, f(1) = negative

So; f(x) has one root in [0, 1]

74. $\sin^2\theta + \text{cosec}^2\theta = (\sin\theta + \text{cosec}\theta)^2 - 2\sin\theta\,\text{cosec}\theta$

$= (2)^2 - 2 = 4 - 2 = 2$, since $(\sin\theta + \text{cosec}\theta) = 2$.

75. Since $\sin 190° = -\sin 10°$, $\sin 200° = -\sin 20°$,

$\sin 210° = -\sin 30°$, $\sin 360° = \sin 180° = 0$ etc.